Mark Scheme – FINAL

72137

Pearson Edexcel International Advanced Level
in Pure Mathematics P1 (WMA11)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e.if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper or ag- answer given
   - ⎿ or d… The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by ‘MR’ in the body of the script.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

   $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to *x* = …

   $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to *x* = …

2. Formula

   Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

   Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to *x* = …

Method marks for differentiation and integration:

1. Differentiation

   Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

   Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values,but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

WMA11 Pure Mathematics P1
Mark Scheme

Question Number	Scheme	Marks
1(a)	$\left(\frac{dy}{dx}=\right) ...x^2+...x+...x^{-2}$	M1
	$\left(\frac{dy}{dx}=\right) \frac{3}{4}x^2-2x-17x^{-2}$	A1A1
		(3)
(b)	$\left(\frac{dy}{dx}=\right) \frac{3}{4}(2)^2-2(2)-17(2)^{-2}=-\frac{21}{4}$	M1
	$y-\frac{13}{2}="-\frac{21}{4}"(x-2)$	dM1
	$21x+4y-68=0$	A1
		(3)
		(6 marks)

(a)

M1 Reduces the power by 1 on any of the following terms $...x^3\to...x^2$, $...x^2\to...x^1$, $...x^{-1}\to...x^{-2}$. Be careful not to allow just sight of x^2. The index does not have to be processed for this mark.

A1 Two of $+\frac{3}{4}x^2, -2x, -17x^{-2}$ or exact unsimplified equivalent terms. Accept eg $\frac{-17}{x^2}$ but the indices must be processed. Double signs eg $+\frac{-17}{x^2}$ is fine.

A1 $\frac{3}{4}x^2-2x-17x^{-2}$ all on one line or exact simplified equivalent. Allow x^1. Withhold the final mark if they attempt to multiply all the terms by 4 for example or a $+c$ appears.

(b)

M1 Attempts to substitute $x=2$ into their $\frac{dy}{dx}$ which must be a changed function to find the gradient of the tangent at P

dM1 It is for the method of finding a line passing through $\left(2, \frac{13}{2}\right)$ using their $\frac{dy}{dx}$ at $x=2$ eg it cannot be the gradient of the normal.

Score for sight of $y-\frac{13}{2}="-\frac{21}{4}"(x-2)$ with both coordinates substituted in correctly or if they use the form $y=mx+c$ they must proceed as far as $c=\dots$ It is dependent on the previous method mark.

A1 $21x+4y-68=0$ or any equivalent multiple where the coefficients are integers and all terms are on one side of the equation. If they state values which contradict the equation then the equation takes precedence.

Question Number	Scheme	Marks
2(a)	$a = 2$	B1
	$b = -3$	B1
		(2)
(b)	Any two term of $\int \frac{2x^3 - 3x^2 - 32x - 15}{5\sqrt{x}}\,dx = \int \frac{2}{5}x^{\frac{5}{2}} - \frac{3}{5}x^{\frac{3}{2}} - \frac{32}{5}x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}\,dx$	M1A1
	$x^n \to x^{n+1}$	M1
	$\frac{4}{35}x^{\frac{7}{2}} - \frac{6}{25}x^{\frac{5}{2}} - \frac{64}{15}x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$	A1A1
		(5)
		(7 marks)

(a) If there is a contradiction between their stated values and the embedded values in a cubic expression then the embedded values take precedence.

B1 $a = 2$ which may be embedded

B1 $b = -3$ which may be embedded

(b)

M1 Attempts to write as a sum of terms. Award for any term with a correct index from correct working. The index does not need to be processed.

A1 Any **two** correct unsimplified or simplified terms of the expression $\frac{2}{5}x^{\frac{5}{2}} - \frac{3}{5}x^{\frac{3}{2}} - \frac{32}{5}x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$ oe eg $0.4x^{\frac{5}{2}} - 0.6x^{\frac{3}{2}} - 6.4x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$ (indices must be processed)

M1 Increases the power of any of their non-integer terms by 1. ($x^n \to x^{n+1}$) This mark cannot be awarded for just increasing the power by 1 on a numerator or denominator term. The index does not need to be processed.

A1 Any two terms correct unsimplified or simplified (see below). May appear in a list or on separate lines.

A1 $\frac{4}{35}x^{\frac{7}{2}} - \frac{6}{25}x^{\frac{5}{2}} - \frac{64}{15}x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$ all on one line (including the constant c and all simplified)

Allow exact equivalents but not rounded decimals so $\frac{4}{35}$ must be written as a fraction.

Accept $\frac{4}{35}x^{\frac{7}{2}} - 0.24x^{\frac{5}{2}} - 4.2\dot{6}x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$. Also allow terms written as eg $\frac{4}{35}x^3\sqrt{x}$ Withhold this mark if the final answer has the integral sign and dx around it or any other spurious notation.

Question Number	Scheme	Marks
3(a)	$1.05 = 3p + q$ $1.65 = 5p + q$ e.g. $\Rightarrow 2p = 0.6$ or $\Rightarrow 1.05 = 3p + (1.65 - 5p)$ or $\Rightarrow 1.65 = 5p + (1.05 - 3p)$ $p = 0.3 \quad q = 0.15$	 M1 A1A1
		(3)
(b)	$2.5 = "0.3"T + "0.15" \Rightarrow T =$ $T = 7.8$	M1 A1
		(2)
		(5 marks)

(a)

M1 Forms two simultaneous equations (which may be in pence) and proceeds to find a value for p or q
May be implied by a correct p or q (allow sight of 30 or 15).
Also score for an attempt to calculate $\frac{1.65 - 1.05}{5 - 3}$ or equivalent.

A1 $p = 0.3$ or $q = 0.15$ oe

A1 $p = 0.3$ and $q = 0.15$ oe

(b)

M1 Uses their values for p and q with $V = 2.5$ and rearranges to find a value for T. It must come from $\frac{2.5 \pm q}{p}$ which you may need to check on your calculator, allowing for truncation or rounding (eg 8 following correct values for V, p and q)

A1 7.8 cao isw after sight of the correct answer.

Question Number	Scheme	Marks
4(a)	$x^2(2x+1)-15x \Rightarrow 2x^3+x^2-15x = x(2x^2+x-15)$	M1
	$x(2x-5)(x+3)=0 \Rightarrow x=...$	dM1
	Two of $x=0,\ \frac{5}{2},\ -3$	B1
	$x=0,\ \frac{5}{2},\ -3$	A1
		(4)
(b)	$y^{\frac{2}{3}}=\frac{5}{2} \Rightarrow y=\left(\frac{5}{2}\right)^{\frac{3}{2}}$	M1
	$\frac{5}{4}\sqrt{10}$	A1cso
		(2)
		(6 marks)

The question says "In this question you must show detailed reasoning. Solutions relying on calculator technology are not acceptable". Correct answers do not imply full marks.

(a)

M1 Multiplies out the bracket to achieve a cubic, takes out a linear factor or cancels the x leading to a quadratic factor (usually $2x^2+x-15$). Award for $...x(...x^2 \pm ...x \pm ...)$ or $...x(...x\pm...)(...x\pm...)$ May be implied by eg $x(2x-5)(x+3)$ May see $=0$ or may be implied.

dM1 Attempts to solve their quadratic either by factorising or using the quadratic formula or completing the square. They cannot just state the roots and the fully factorised version or values of a, b and c must match their quadratic. It is dependent on the previous method mark. The = 0 can be implied and the factorised quadratic may appear under their solutions.

B1 Two of $x=0,\ \frac{5}{2},\ -3$

A1 $x=0,\ \frac{5}{2},\ -3$ provided all previous marks have been scored. Check for $x=0$ in earlier work

Eg1 $2x^3+x^2-15x = x(2x-5)(x+3) \Rightarrow 0,\ \frac{5}{2},\ -3$ M1dM1B1A1

Eg2 $2x^3+x^2-15x=0 \Rightarrow x^3+\frac{1}{2}x^2-\frac{15}{2}x=0 \Rightarrow x\left(x-\frac{5}{2}\right)(x+3)=0 \Rightarrow 0,\ \frac{5}{2},\ -3$ M1dM1B1A1

Eg3 $2x^3+x^2-15x = x\left(x-\frac{5}{2}\right)(x+3) \Rightarrow 0,\ \frac{5}{2},\ -3$ M1dM0B1A0 (the cubic and factorised version are not equal to each other)

Eg4 $2x^3+x^2-15x = x(2x^2+x-15) \Rightarrow 0,\ \frac{5}{2},\ -3$ M1dM0B1A0 (no method seen to solve the quadratic)

Eg5 $0,\ \frac{5}{2},\ -3$ M0dM0B1A0 (no method seen at all)

This response below can score full marks.

(a). $f(x) = x^2(2x+1) - 15x$.

$0 = 2x^3 + x^2 - 15x$

$(2x-5)(x+3)(x+0)$.

$x = \frac{5}{2}$ $x = -3$ $x = 0$.

(b)

M1 $y^{\frac{2}{3}} = "\frac{5}{2}" \Rightarrow y = \left("\frac{5}{2}"\right)^{\frac{3}{2}}$ (or one of their positive solutions from part (a)). The question states a calculator cannot be used so do not award for an implied method if they have decimal solutions or the exact answer without the method to deal with the fractional power seen. No marks can be scored without a positive solution from (a) or they restart in (b).

Allow notation such as $y = \sqrt[\frac{2}{3}]{\frac{5}{2}}$ and you do not need to see $y = ...$

A1 $\frac{5}{4}\sqrt{10}$ or $1.25\sqrt{10}$ or $1\frac{1}{4}\sqrt{10}$ **only and no other solutions** cso

Eg1 $y^{\frac{2}{3}} = "\frac{5}{2}" \Rightarrow y = 3.95...$ is M0A0

Eg2 $y^{\frac{2}{3}} = "\frac{5}{2}" \Rightarrow y = \frac{5}{4}\sqrt{10}$ is M0A0

Eg3 $y^{\frac{2}{3}} = "\frac{5}{2}" \Rightarrow y^2 = \frac{125}{8} \Rightarrow y = \frac{5}{4}\sqrt{10}$ can score M1A1 as they have shown a method to deal with the factional power.

Question Number	Scheme	Marks
5(a)	$\mathrm{f}'(x)=12x^{-\frac{1}{2}}+\frac{x}{3}-4$	
	One of $x^{-\frac{1}{2}}\to x^{\frac{1}{2}}$, $-4\to -4x$, $x\to x^2$	M1
	$\mathrm{f}(x)=\int 12x^{-\frac{1}{2}}+\frac{x}{3}-4\,\mathrm{d}x=24x^{\frac{1}{2}}+\frac{x^2}{6}-4x+c$	A1A1
	$8=24(9)^{\frac{1}{2}}+\frac{(9)^2}{6}-4(9)+c\Rightarrow c=...$	dM1
	$(\mathrm{f}(x)=)\ 24x^{\frac{1}{2}}+\frac{x^2}{6}-4x-\frac{83}{2}$	A1
		(5)
(b)	$\mathrm{f}'(9)=\frac{12}{\sqrt{9}}+\frac{9}{3}-4\quad(=3)$	M1
	$3\to -\frac{1}{3}$	dM1
	$y-8=\text{"}-\frac{1}{3}\text{"}(0-9)$	M1
	$(0, 11)$	A1
		(4)
		(9 marks)

(a)

M1 Integrates by raising the power on one of the terms (ie $x^{-\frac{1}{2}}\to x^{\frac{1}{2}}$, $-4\to -4x^1$, $x\to x^2$). The index does not need to be processed.

A1 Two terms correct of $24x^{\frac{1}{2}}+\frac{x^2}{6}-4x$ or unsimplified equivalent which may appear as a list. The indices must be processed. Allow x^1

A1 $24x^{\frac{1}{2}}+\frac{x^2}{6}-4x\ (+c)$ or unsimplified equivalent. Condone the lack of +*c* for this mark. Allow x^1 and ignore any spurious notation including = 0

dM1 Attempts to substitute $x=9, y=8$ into their f(*x*) and proceeds to find *c*. It is dependent on the previous method mark. If they have no + *c* then this mark cannot be scored.

A1 $(\mathrm{f}(x)=)\ 24x^{\frac{1}{2}}+\frac{x^2}{6}-4x-\frac{83}{2}$ or simplified equivalent. Withhold this mark if they attempt to make the coefficients integers. Do not accept rounded decimals such as $0.166x^2$. Also withhold this mark if spurious notation around the answer is seen such as the integral and d*x* or an = 0

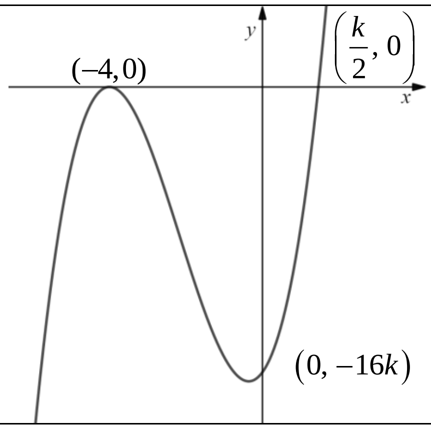
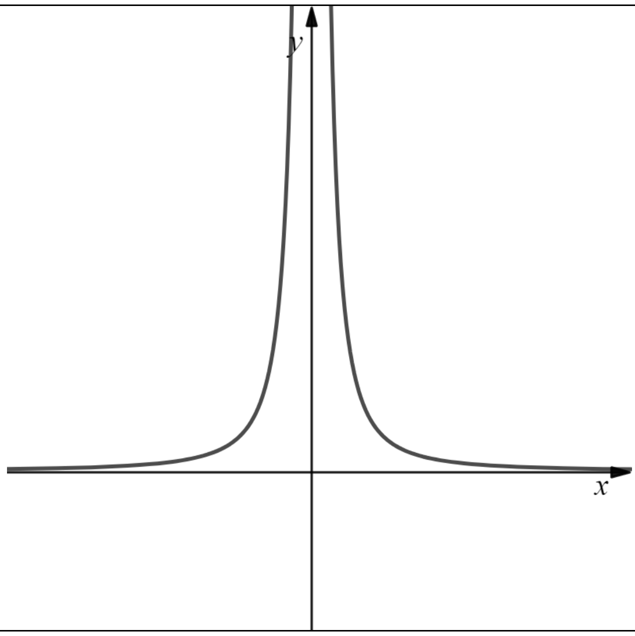
(b)

M1 Attempts to substitute $x = 9$ to find a value for $f'(9)$

dM1 Attempts to find the negative reciprocal to find the gradient of the normal. It is dependent on the previous method mark.

M1 It is for the method of finding a line passing though $(9, 8)$, using a changed gradient and substituting $x = 0$. Alternatively, they may just use their gradient to determine where the line crosses the *y*-axis knowing that for every "3 units to the left it is 1 up".

A1 $(0, 11)$ or allow it to be written as $x = 0, y = 11$

Question Number	Scheme	Marks
6(a)(i)		B1B1B1
		(3)
(a)(ii)		B1B1
		(2)
(b)	One root because the two graphs intersect each other once	B1
		(1)
		(6 marks)

(a)(i) If multiple attempts are made then mark the last attempt. Condone if (i) and (ii) are drawn on the same set of axes

B1 A positive cubic shape with a local maximum and a local minimum drawn anywhere on a set of axes. Do not be concerned regarding the location of the minimum. Mark of the intention to draw a cubic so condone aspects which may appear linear or slips of the pen.

B1 Intersects (not just meet) the *x*-axis at $x = \frac{k}{2}$ to the right of the *y*-axis and has a local maximum (or minimum) on $x = -4$ which must be to the left of the *y*-axis. If there is a contradiction between the labelling on the graph and values stated separately then the graph labels take precedence. Condone for example $(-4, 0)$ labelled as $(0, -4)$ as long as it is in the correct position on the axis

B1 *y*-intercept is $-16k$ (which must be below the *x*-axis) If there is a contradiction between the labelling on the graph and the *y*-intercept stated separately then the graph label takes precedence. Condone for example $(0, -16k)$ labelled as $(-16k, 0)$ as long as it is in the correct position on the axis

(a)(ii)

B1 Correct shaped curve in the first **or** second quadrant. It must not cross either axis. Do not be concerned by any labelled asymptotes or extra “branches” appearing in these quadrants. Mark the intention to draw a graph which does not have a clear turning point.

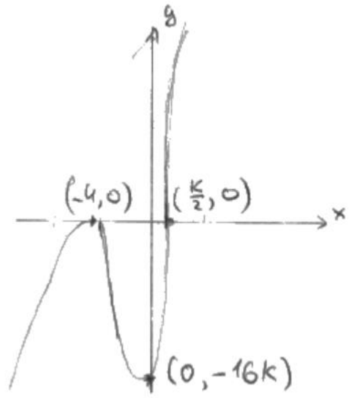
B1 Correct curves in the first **and** second quadrants and no other curves appearing in quadrants 3 and 4 (unless the cubic from (i) has been drawn on the same axes). Mark the intention to draw a graph tending towards the axes and does not have a clear turning point. If any asymptotes are labelled or indicated with dashed lines which are not the coordinate axes then withhold this mark.

(b) **This mark can only be awarded provided both graphs are the correct shape and position in part (a) (the intercepts with the axes are not needed for this mark)**

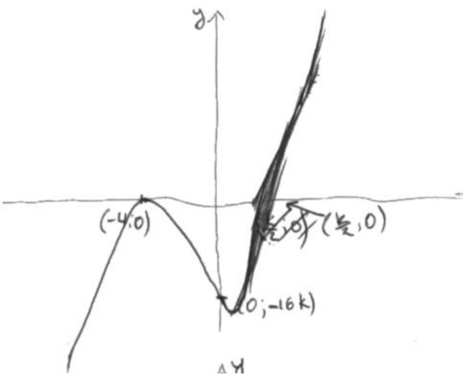
B1 One root because the two graphs intersect each other once (only). Condone alternative wording which implies they “meet” once. Do not allow references to intersecting the axes.

Examples for (a)(i)

B1B1B1

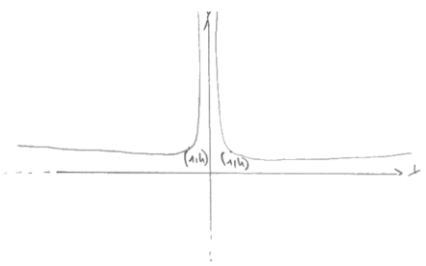


B1B1B1

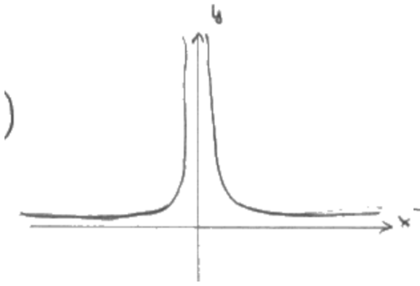


Examples for (a)(ii)

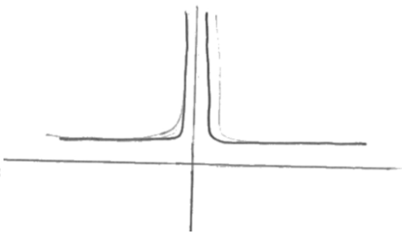
B1B1



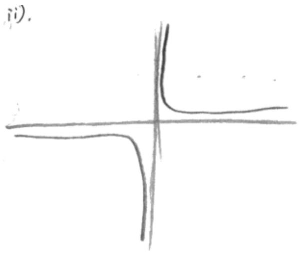
B1B1



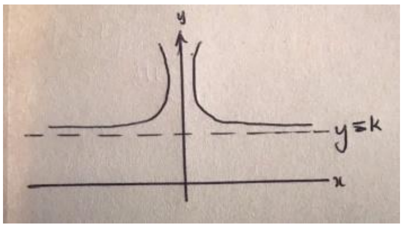
B1B1



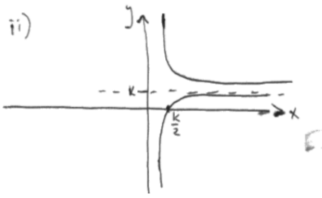
B1B0

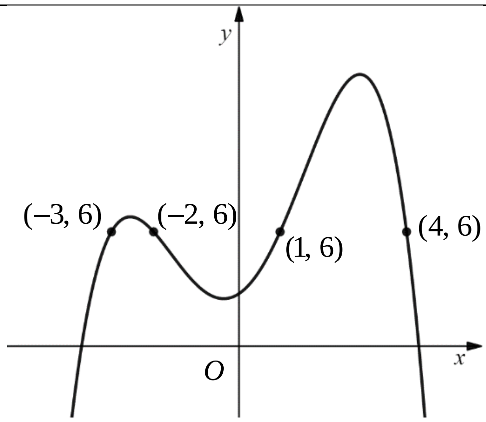


B1B0



B1B0



Question Number	Scheme	Marks
7(a)	$-1 < x < 2$ $x < -4,\ x > 3$	M1A1 B1
		(3)
(b)	$(x =)\ 1.5$	B1
		(1)
(c)(i) **(ii)**	 $-3 \leqslant x \leqslant -2$	B1B1B1 B1
		(4)
		(8 marks)

(a) Ignore any references to OR or AND (or equivalent use of set notation) between $-1 < x < 2,\ x > 3,\ x < -4$

M1 Requires one end of the inside region between -1 and 2. Score for $x > -1$ or $x < 2$, condoning use of $\leqslant$ $\geqslant$ Must be in terms of x

A1 $-1 < x < 2$ or any equivalent notation such as $-1 < x \cap x < 2$, $(-1, 2)$ and also score for $2 > x > -1$. Do not allow just two separate inequalities for this mark without AND or equivalent.

B1 $x > 3,\ x < -4$

(b)

B1 1.5 or $\frac{3}{2}$ Condone $(1.5,\ 6)$ or $x = 1.5,\ y = 6$ only (the y-coordinate must be correct if stated)

(c)(i) **Check the figure at the start of the question. If there is a contradiction, then the sketch with labelled coordinates in the main body of working takes precedence. There must be a sketch to score any marks.**

B1 Sketch reflected in the y-axis with the intention to have the higher maximum turning point in quadrant 1 and the lower maximum turning point in quadrant 2. Do not be concerned which side of the y-axis the minimum turning point is located.

B1 Either two correct pairs of coordinates **or** all x coordinates correct **or** all y coordinates correct. Do not be concerned with the relative locations of the coordinates in relation to each other. The points must be in the correct quadrants, but may be stated separately. Do not be concerned with the labelling of P, Q, R and S and the associated coordinates if they are stated separately.

B1 All four correct coordinates (Do not penalise poor notation and may be listed as $x = ..., \ y = ...$) Do not be concerned with the relative locations of the coordinates in relation to each other. The points must be in the correct quadrants, but may be stated separately. Do not be concerned with the labelling of P, Q, R and S and the associated coordinates if they are stated separately.

(ii)

B1 $-3 \leqslant x \leqslant -2$ only or any equivalent notation such as $-3 \leqslant x \cap x \leqslant -2$,
$-3 \leqslant x$ AND $x \leqslant -2$. [–3, 2] but do not accept use of OR or $\cup$ if set notation is used.
Do not accept $(-3, -2)$ or $-3 < x < -2$ Must be in terms of x
There may be several inequalities. Mark what appears to be their final answer.

Question Number	Scheme	Marks
8(a)(i)	$2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$	B1
	Area of sector $= \frac{1}{2}\times 3^2 \times "\frac{4}{3}\pi" = 6\pi$ (m^2)	M1A1
(ii)	Length of arc $= 3\times "\frac{4}{3}\pi" \Rightarrow$ Perimeter $= 4\pi + 6$ (m)	M1A1
		(5)
(b)	$\frac{1}{2}\times 3^2 \times \sin\left(\frac{2}{3}\pi\right) = \frac{9\sqrt{3}}{4}$ (m^2)	M1A1
		(2)
(c)	Eg $AB^2 = 3^2 + 3^2 - 2\times 3\times 3\times \cos\left(\frac{2}{3}\pi\right) \Rightarrow AB^2 = 27 \Rightarrow AB = 3\sqrt{3}$ (m) *	M1A1*
		(2)
(d)	$\frac{\sin BAC}{8} = \frac{\sin\left(\frac{\pi}{6}\right)}{3\sqrt{3}} \Rightarrow \sin BAC = ...$ or $BAC = ...$	M1
	$\sin BAC =$ awrt $\frac{4\sqrt{3}}{9}$ or $BAC =$ awrt 0.88 $(0.8785\ldots)$	A1
	Area $ABC = \frac{1}{2}\times 3\sqrt{3}\times 8\times \sin(\pi - \frac{\pi}{6} - "0.88")$ $(= 20.4896....)$	M1
	Total area = "18.8"+"3.90"+"20.5"	dM1
	= awrt 43 (m^2)	A1
		(5)
		(14 marks)

If lengths or areas are found in other parts then credit can be awarded for these as long as they are referred to or used in the relevant part.

(a)(i)

B1 Finds the correct angle for the sector *AOBX*. They may find the minor sector first and then subtract from the area of the whole circle so may be implied from later work. They may also work in degrees. Sight of $\theta = \frac{4\pi}{3}$ on the diagram or within their working scores this mark.

M1 States or uses $\frac{1}{2}r^2\theta$ with $r = 3$ and $\theta = \frac{2\pi}{3}$ or $\theta = \frac{4\pi}{3}$ or the equivalent method working in degrees. May be implied by the correct answer, expression or awrt 9.42 or awrt 18.8

A1 6π (m^2) cao must be exact. Isw after a correct answer.

(ii)

M1 States or uses $r\theta$ with $r = 3$ and $\theta = \frac{2\pi}{3}$ or $\theta = \frac{4\pi}{3}$ or the equivalent method working in degrees. The addition of two radii to find the perimeter is not required for this mark. May be implied by the correct answer, expression or awrt 6.28 or awrt 12.6

A1 $4\pi + 6$ (m) cao must be exact. Isw after a correct answer.

(b)

M1 States or uses $\frac{1}{2}ab\sin C$ with $a = b = 3, \theta = \frac{2}{3}\pi$ (or may work in degrees). Alternatively, they may split the isosceles triangle into two right angled triangles including finding *AB*. Score for the overall method or awrt 3.90

A1 $\frac{9\sqrt{3}}{4}$ (m^2) or exact equivalent. Isw after a correct answer

(c)

M1 Correct method (which may be seen in earlier work but referred to in (c)) by for example

- using the cosine rule with $a = b = 3, \theta = "\frac{2}{3}\pi"$
- splitting the isosceles triangle into two right angled triangles eg $2 \times 3 \times \sin\left(\frac{\pi}{3}\right)$
- using the sine rule such that $\frac{AB}{\sin\left(\frac{2\pi}{3}\right)} = \frac{3}{\sin\left(\frac{\pi}{6}\right)} \Rightarrow AB = ...$

A1* $3\sqrt{3}$ (m) with no errors in their calculations. Minimum expected to see is a simplified expression for AB (or AB^2) which is not $3\sqrt{3}$. Eg via the cosine rule this would be either $AB^2 = 27$ or $AB = \sqrt{27}$ via the sine rule eg $\frac{3\sin(\frac{2\pi}{3})}{\sin(\frac{\pi}{6})}$ or via splitting the triangle up into two right angle triangles $2 \times 3 \times \sin\left(\frac{\pi}{3}\right)$ oe. Accept alternative labelling for AB eg x

You do not need to see the exact values for the the trig functions within their working.

(d) **Beware there are many methods with incorrect working leading to 43 so you will need to check carefully that their method is sound.**

M1 Attempts to use the sine rule to find sin BAC or angle BAC. Award for the appropriate lengths and angles in the correct positions within a correct equation. Do not be concerned with the mechanics of the rearrangement, although it must be a solvable equation. Alternatively attempts to solve a quadratic in AC using the cosine rule.

$(3\sqrt{3})^2 = AC^2 + 8^2 - 2 \times AC \times 8 \times \cos\left(\frac{\pi}{6}\right) \Rightarrow AC = ...$ $(= \sqrt{11} + 4\sqrt{3} =$ awrt 10.2)

A1 Correct value for $\sin BAC$, angle BAC (awrt 0.88 or 50.3 in degrees) or AC

M1 Correct method to find the area of triangle ABC using either their angle ABC from $\pi - \frac{\pi}{6} - "BAC"$ or the length AC. May work in degrees.

dM1 Adds their (a)(i) + their (b) + their triangle ABC together to the find area of the pond. It is dependent on all of the previous method marks in (d) only

A1 awrt 43 (m^2) from a correct method. Also allow the exact answer $6\pi + 2\sqrt{11} + \frac{41}{4}\sqrt{3}$ oe

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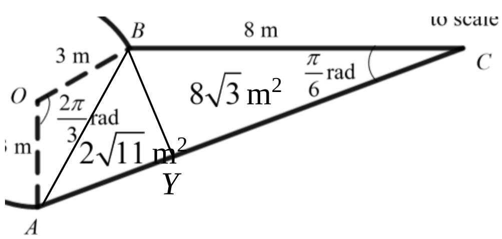
Alt (d) Forms two right angled triangles with the perpendicular to side AC from point B



M1 Height of the triangle ABC is $BY = 8\sin\left(\frac{\pi}{6}\right)$

A1 $BY = 4$ may be implied

M1 Attempts to find the area of triangle ABC. Usually this is by attempting to find length AC split into AY and YC eg $AY = \sqrt{\left(3\sqrt{3}\right)^2 - "4"^2}$ $\left(= \sqrt{11}\right)$ and length $YC = 8\cos\left(\frac{\pi}{6}\right) = 4\sqrt{3}$

$\Rightarrow$ Area $ABC = \frac{1}{2} \times \left(\sqrt{11} + 4\sqrt{3}\right) \times 4$. Score for the overall method condoning slips.

Alternatively, they may attempt the cosine rule to find $AC \Rightarrow$ Area $ABC = ...$

dM1A1 Follows main scheme

Question Number	Scheme	Marks
9(a)	$\frac{1}{2}x^2-10x+22=\frac{1}{2}(x\pm...)^2\pm...$ or states $a=\frac{1}{2}$	B1
	$\frac{1}{2}x^2-10x+22=\frac{1}{2}(x\pm10)^2\pm...$ or states $a=\frac{1}{2}$ and $b=\pm10$	M1
	$\frac{1}{2}x^2-10x+22=\frac{1}{2}(x-10)^2-28$	A1
		(3)
(b)	$("10", "-28")$	B1ftB1ft
		(2)
(c)(i)	Gradient of tangent $=8$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=x-10=8\Rightarrow x=...$	M1
	$x=18,\ y=4$	A1A1
(c)(ii)	$k-\frac{1}{8}\times"18"="4"\Rightarrow k=\frac{25}{4}$	M1A1
		(6)
(d)	One of $x\geqslant"10"$ or $y\leqslant"\frac{25}{4}-\frac{1}{8}x"$ or $y\geqslant\frac{1}{2}x^2-10x+22$	B1ft
	Two of $x\geqslant"10"$ or $y\leqslant"\frac{25}{4}-\frac{1}{8}x"$ or $y\geqslant\frac{1}{2}x^2-10x+22$	B1ft
	All three of $x\geqslant10$, $y\leqslant\frac{25}{4}-\frac{1}{8}x$ and $y\geqslant\frac{1}{2}x^2-10x+22$	B1
		(3)
		(14 marks)

(a) **If there is a contradiction between the embedded values of *a*, *b* and *c* within their expression and their stated values then the embedded values take precedence.**

B1 Achieves $\frac{1}{2}x^2-10x+22=\frac{1}{2}(x\pm...)^2\pm...$ or states that $a=\frac{1}{2}$

M1 Deals correctly with the first two terms of $\frac{1}{2}x^2-10x+22$

Scored for $\frac{1}{2}x^2-10x+22=\frac{1}{2}(x\pm10)^2\pm...$ or states that $a=\frac{1}{2}$ & $b=\pm10$ (may be found using symmetry by finding the roots)

A1 $\frac{1}{2}x^2-10x+22=\frac{1}{2}(x-10)^2-28$ (cannot just be the stated values). Do not isw if for example they attempt to multiply by 2 to achieve integer values

This may also be done by equating coefficients using the expanded form $a(x+b)^2+c=ax^2+2abx+ab^2+c$ but the marks can be applied in the same way

(b)

B1ft One of the coordinates of $(10, -28)$ or follow through one of their $(-b, c)$ from (a).

B1ft $(10, -28)$ or follow through their $(-b, c)$ from (a). Accept $x = "10",\ y = "-28"$. Condone lack of bracketing as long as the intention as to which coordinate is which is clear.

(c)(i)

B1 Gradient of tangent $= 8$ (stated or implied)

M1 Differentiates $\frac{1}{2}x^2 - 10x + 22$ to achieve a derivative of the form $px + q$ and sets equal to 8. They then proceed to find a value for *x*.

Alternatively, sets $8x + \alpha = \frac{1}{2}x^2 - 10x + 22$, proceeds to a quadratic in *x*, sets the discriminant = 0 and solves to find α. They then use this value of α in their original equation to solve and find *x*

A1 $x = 18$

A1 $y = 4$

(c)(ii)

dM1 Sets $k - \frac{1}{8} \times "18" = "4"$ and proceeds to find a value for *k*. It is dependent on the method mark in (i)

A1 $k = \frac{25}{4}$ oe

(d) **Ignore any references to OR or AND (or equivalent use of set notation)**

Note that $\frac{1}{2}x^2 - 10x + 22 \leqslant y \leqslant "\frac{25}{4}" - \frac{1}{8}x$ is acceptable and counts as two of the required inequalities in part (d)

Use of strict or inclusive inequalities must be consistent for all of their inequalities on the last mark only

B1ft One of $x \geqslant "10"$ (or $"10" \leqslant x \leqslant d$ where *d* is at least 18) or $y \leqslant "\frac{25}{4}" - \frac{1}{8}x$ or $y \geqslant \frac{1}{2}x^2 - 10x + 22$ follow through on their minimum point or their *k*. Allow if *l* is still in terms of *k*

B1ft Two of $x \geqslant "10"$ (or $10 \leqslant x \leqslant d$ where *d* is at least 18) or $y \leqslant "\frac{25}{4}" - \frac{1}{8}x$ or $y \geqslant \frac{1}{2}x^2 - 10x + 22$ follow through on their minimum point or their *k*. Allow if *l* is still in terms of *k*

B1 All three of $x \geqslant 10$ (or $10 \leqslant x \leqslant d$ where *d* is at least 18), $y \leqslant \frac{25}{4} - \frac{1}{8}x$ and $y \geqslant \frac{1}{2}x^2 - 10x + 22$. Condone the additional minimum interval (or greater) $-28 \leqslant y \leqslant 5$